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The effect of temperature-dependent properties on generalized magneto-thermo-elastic medium with two-temperature under three-phase-lag model

Mohamed I.A. Othman
Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt
Yassmin D. Elmaklizi
Department of Mathematics, Faculty of Science, Suez Canal University, Suez, Egypt, and
Nehal T. Mansoure
Department of Mathematics, Faculty of Science, Zagazig University, Zagazig, Egypt

Abstract

Purpose – The purpose of this paper is to investigate the propagation of plane waves in an isotropic elastic medium under the effect of rotation, magnetic field and temperature-dependent properties with two-temperatures.
Design/methodology/approach – The problem has been solved analytically by using the normal mode analysis.
Findings – The numerical results are given and presented graphically when mechanical and thermal force are applied. Comparisons are made with the results predicted by the three-phase-lag (3PHL) model and dual-phase-lag model in the presence and absence of cases where the modulus of elasticity is independent of temperature.
Originality/value – In this work, the authors study the influence of rotation and magnetic field with two-temperature on thermoelastic isotropic medium when the modulus of elasticity is taken as a linear function of reference temperature in the context of the 3PHL model. The numerical results for the field quantities are obtained and represented graphically.

Keywords Magnetic field, Rotation, Thermoelastic, Thermodynamic temperature, Three-phase-lag, Two-temperature

Paper type Research paper

Nomenclature

λ, μ	Lame' parameters	b	constant material "two temperature parameter"
δ_{ij}	Kronecker delta	u_i	displacement vector
p	initial stress	ρ	mass density
a	the volume coefficient of thermal expansion	c_e	specific heat at constant strain
K^*	material characteristic of the theory	$K \geqslant 0$	thermal conductivity
T	thermodynamic temperature	T_0	reference temperature
τ_v	phase lag of thermal displacement gradient	ϕ	conductive temperature
		τ_t	phase lag of temperature gradient
τ_q	phase lag of heat flux	h	induced magnetic field vector
ε_0	electric permeability for free space	μ_0	magnetic permeability for free space
E	induced electric field vector	J	current density vector
H_0	initial uniform magnetic field	F_i	Lorentz force

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1. Introduction

The generalized theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity. Hetnarski and Ignaczak (1999) examined five generalizations of the coupled theory of thermoelasticity. Lord and Shulman (1967) formulated the first generalization of the generalized thermo-elasticity theory involving one thermal relaxation time. Green-Lindsay (1972) developed the temperature rate-dependent thermoelasticity, which includes two thermal relaxation times and does not violate the classical Fourier's law of heat conduction, when the body under consideration has a center of symmetry. One can review the presentation of generalized theories of thermoelasticity by Hetnarski and Ignaczak (1996). The wave propagation in anisotropic solids in generalized theories of thermoelasticity was studied by Sharma and Sidhu (1986). The third generalization of the coupled theory of thermoelasticity is developed by Hetnarski and Ignaczak (1996) and is known as low-temperature thermoelasticity. The fourth generalization to the coupled theory of thermoelasticity introduced by Green and Naghdi and this theory is concerned with the thermoelasticity theory without energy dissipation, referred to as G-N theory of type II in which the classical Fourier law is replaced by a heat flux rate-temperature gradient relation and Green and Naghdi with energy dissipation referred to as G-N theory of type III. The fifth generalization of the coupled theory of thermoelasticity is referred to the dual-phase-lag (DPL) thermoelasticity by Tzou (1995a) and Chandrasekharaiah (1998). For macroscopic formulation, it would be convenient to use the DPL mode for investigation of the microstructural effect on the behavior of heat transfer. The physical meanings and the applicability of the DPL mode have been supported by the experimental results in Tzou (1995b). Recently, the three-phase-lag (3PHL) heat conduction equation in which the Fourier's law of heat conduction is replaced by an approximation to a modification of the Fourier law with the introduction of three different phases-lags for the heat flux vector, the temperature gradient and the thermal displacement gradient by Roy Choudhuri (2007). The stability of the 3PHL heat conduction equation was discussed by Quintanilla and Racke (2008). Subsequently, this theory has employed thermoelasticity with a 3PHL model to discuss a problem of thermoelastic interactions on functional graded orthotropic hollow sphere under thermal shock by Kar and Kanoria (2009).

Some researcher in the past investigated different problems of rotating media. Schoenberg and Censor (1973) studied the propagation of plane harmonic waves in a rotating elastic medium without a thermal field. It was shown there that the rotation causes the elastic medium to be depressive and anisotropic. The effect of magnetic field on the 2D problem of thermoelastic fiber-reinforced medium under temperature-dependent properties with the 3PHL model was presented by Othman and Said (2014). The effect of rotation on elastic waves has been studied by Clarke and Burdess (1994) and Destrade (2004). Othman and Song (2009) discussed the effect of rotation in a magneto-thermoelastic medium. Othman *et al.* (2015) studied the effect of gravity and rotation on microstretch thermoelastic medium with diffusion of the DPL model.

The two-temperature theory of thermoelasticity was introduced by Gurtin and Williams (1967) and Chen and Gurtin (1968) in which the classical Clausius-Duhem inequality was replaced by another one depending on two-temperature: the conductive temperature and the thermodynamic temperature, the first is due to the thermal processes, and the second is due to the mechanical processes inherent between the particles and the layers of elastic material; this theory was also investigated by Ieşan (1970). The two-temperature model was underrated and unnoticed for many years thereafter. Only in the last decade the theory has been noticed, developed in many searches, and found its applications, mainly in the problems in which the discontinuities of stresses have no physical interpretations. Among the authors who contribute to develop this theory, Quintanilla (2004) studied the existence, structural stability, convergence and spatial behavior of this theory. Youssef (2006) introduced the generalized Fourier law to the field equations of the two-temperature theory of thermoelasticity and proved the uniqueness of the solution for the homogeneous isotropic material. The effect of two-temperature and gravity on the 2D problem of thermo-viscoelastic material under the 3PHL model has been studied by Othman and Zidan (2015).

Some researchers investigated different problems of the magnetic field. Othman *et al.* (2015) studied the effect of magnetic field on generalized thermoelastic medium with two-temperature under the 3PHL model. Othman and Hilal (2016) studied the propagation of plane waves of magneto-thermoelastic medium with voids influenced by the gravity and laser pulse under G-N theory. The effect of rotation and modified Ohm's law influence on magneto-thermoelastic micropolar material with microtemperatures has been studied by Othman *et al.* (2016). In most of the problems, the material properties of the medium are taken to be constant. However, it is well known that the physical properties of engineering materials vary considerably with temperature. Othman (2002, 2003) used the normal mode analysis and state-space approach to study two-dimensional problems of generalized thermoelasticity with one and two relaxation times where the modulus of elasticity depends on the reference temperature.

The aim of this paper is to study the influence of rotation and magnetic field with two-temperature on thermoelastic isotropic medium when the modulus of elasticity is taken as a linear function of reference temperature in the context of the 3PHL model. The numerical results for the field quantities are obtained and represented graphically. Comparisons are made with the results predicted by the 3PHL model and DPL model in the presence and absence of the case where the modulus of elasticity is independent of temperature.

2. Formulation of the problem

We consider an isotropic, homogeneous, linear and thermally elastic medium with temperature-dependent mechanical properties. The basic equations for a homogeneous isotropic thermoelastic solid under the influence of magnetic field rotate uniformly with angular velocity $\boldsymbol{\Omega} = \Omega \boldsymbol{n}$, where $\boldsymbol{n}$ is a unit vector representing the direction of the axis of rotation. All quantities are considered functions of the time variable t and of the coordinates x and y. The displacement equation in the rotating frame has two additional terms (Schoenberg and Censor, 1973): centripetal acceleration $\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{u})$ due to time varying motion only and Coriolis acceleration $2\Omega \times \dot{u}$ where $\boldsymbol{u} = (u_1,\ u_2,\ 0)$ is the dynamic displacement vector and angular velocity is $\boldsymbol{\Omega} = (0, 0, \Omega)$. These terms do not appear in non-rotating media. A magnetic field with constant intensity $\boldsymbol{H} = (0, 0, H_0)$ acts parallel to the bounding plane (take as the direction of the z-axis). Application of initial magnetic field $\boldsymbol{H}$ results in an induced magnetic field $\boldsymbol{h}$ and an induced electric field $\boldsymbol{E}$. The simplified linear equations of electrodynamics of slowly moving medium for a homogeneous, thermally and electrically conducting elastic solid are:

$$curl\ \boldsymbol{h} = \boldsymbol{J} + \varepsilon_0 \dot{\boldsymbol{E}}, \tag{1}$$

$$curl\ \boldsymbol{E} = -\mu_0 \dot{\boldsymbol{h}}, \tag{2}$$

$$div\ \boldsymbol{h} = 0, \tag{3}$$

$$\boldsymbol{E} = -\mu_0 (\dot{\boldsymbol{u}} \times \boldsymbol{H}). \tag{4}$$

The stress-strain relation:

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e - \gamma (T - T_0)]\delta_{ij}. \tag{5}$$

The displacement components have the following form $\boldsymbol{u} = (u,\ v,\ 0)$.

The equations of motion in the absence of body force:

$$\sigma_{j1,j}+F_1=\rho\left[\ddot{\boldsymbol{u}}_1+\{\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u})\}_1+(2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}})_1\right], \tag{6}$$

$$\sigma_{j2,j}+F_2=\rho\left[\ddot{\boldsymbol{v}}_2+\{\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\boldsymbol{u})\}_2+(2\boldsymbol{\Omega}\times\dot{\boldsymbol{u}})_2\right]. \tag{7}$$

The equation of heat conduction under the 3PHL model:

$$K^*\nabla^2\phi+\tau_v^*\nabla^2\dot{\phi}+K\tau_t\nabla^2\ddot{\phi}=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}\right)\left(\rho c_e\ddot{T}+\gamma T_0\ddot{e}\right), \tag{8}$$

where $\tau_v^*=(K+K^*\tau_v)$,

$$F_i=\mu_0(J\times H)_i. \tag{9}$$

$$T=\left(1-b\nabla^2\right)\phi, \tag{10}$$

$$e_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}). \tag{11}$$

We assume that:

$$\lambda=\lambda_0 f(T),\quad \mu=\mu_0 f(T),\quad \gamma=\gamma_0 f(T). \tag{12}$$

where $f(T)$ is a given dimensionless function of temperature such that $f(T)=1-\alpha^*T_0$ (α^* is an empirical material constant). In the case of the temperature independent modulus of elasticity, we have $f(T)=1$. Using (5), (9), and (12), equations of motion (6) and (7) take the form:

$$\mu_0 f(T)\nabla^2u+\left[\lambda_0 f(T)+\mu_0 f(T)+\mu_0H_0^2\right]\frac{\partial e}{\partial x}-\gamma_0 f(T)\left(1-b\nabla^2\right)\frac{\partial\phi}{\partial x}$$
$$-\varepsilon_0\mu_0^2H_0^2\ddot{u}=\rho\left[\ddot{u}-u\Omega^2-2\Omega\dot{v}\right], \tag{13}$$

$$\mu_0 f(T)\nabla^2v+\left[\lambda_0 f(T)+\mu_0 f(T)+\mu_0H_0^2\right]\frac{\partial e}{\partial y}-\gamma_0 f(T)\left(1-b\nabla^2\right)\frac{\partial\phi}{\partial y}$$
$$-\varepsilon_0\mu_0^2H_0^2\ddot{v}=\rho\left[\ddot{v}-v\Omega^2+2\Omega\dot{u}\right], \tag{14}$$

$$K^*\nabla^2\phi+\tau_v^*\nabla^2\dot{\phi}+K\tau_t\nabla^2\ddot{\phi}=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}\right)\left[\left(\rho c_e\left(1-b\nabla^2\right)\ddot{\phi}+\gamma_0 f(T)T_0\ddot{e}\right],\right. \tag{15}$$

where $h=-H_0e$.

For the purpose of numerical evaluation, we introduce the dimensions variables:

$(x',y')=\omega^*/c_0\,(x,y)$, $(u',v')=\rho c_0\omega^*/\gamma_0T_0\,(u,v)$, $\sigma'_{ij}=1/\gamma_0T_0\,\sigma_{ij}$, $[T',\phi']=1/T_0\,[T,\phi]$, $h'=h/H_0$, $(t',\tau'_v,\tau'_q,\tau'_t)=\omega^*(t,\tau_v,\tau_q,\tau_t)$, $c_0^2=\lambda_0+2\mu_0/\rho$, $e=\gamma_0T_0/\rho c_0^2e'$, $\omega^*=\rho c_ec_0^2/K$, $\nabla^2=\omega^{*2}/c_0^2\nabla'^2$ $\gamma=(3\lambda_0+2\mu_0)\alpha_t$, $\Omega'=\Omega/\omega^*$.

Using the above dimensions quantities, then Equations (13)-(15) become:

$$a_1\nabla^2 u+\left[\left(\frac{\mu_0 H_0^2}{\rho c_0^2}+a_1+\left(1-\frac{2}{\beta^2}\right)f(T)\right]\frac{\partial e}{\partial x}\right.$$

$$-(1-b^*\nabla^2)f(T)\frac{\partial \phi}{\partial x}=\left[\left(1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}\right)\ddot{u}-u\Omega^2-2\Omega\dot{v}\right], \tag{16}$$

$$a_1\nabla^2 v+\left[\left(\frac{\mu_0 H_0^2}{\rho c_0^2}+a_1+\left(1-\frac{2}{\beta^2}\right)f(T)\right]\frac{\partial e}{\partial y}\right.$$

$$-(1-b^*\nabla^2)f(T)\frac{\partial \phi}{\partial y}=\left[\left(1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}\right)\ddot{v}-v\Omega^2+2\Omega\dot{u}\right], \tag{17}$$

$$\varepsilon_1\nabla^2\phi+\varepsilon_2\nabla^2\dot{\phi}+\tau_t\nabla^2\ddot{\phi}=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}\right)\left[(1-b^*\nabla^2)\ddot{\phi}+\varepsilon_3\ddot{e}\right]. \tag{18}$$

where $\varepsilon_1=K^*/\rho c_e c_0^2, \varepsilon_2=1+\varepsilon_1\tau_v, \varepsilon_3=\gamma_0^2 T_0 f(T)/\rho^2 c_e c_0^2, b^*=b\omega^{*2}/c_0^2, a_1=\mu_0 f(T)/\rho c_0^2$.

We define the displacement potentials q and ψ which relate to the displacement components u and v as:

$$u=q_{,x}-\psi_{,y},\; v=q_{,y}+\psi_{,x}. \tag{19}$$

Using Equation (19) in Equations (16)-(18), we obtain:

$$\left[\left(\frac{\mu_0 H_0^2}{\rho c_0^2}+2a_1+f(T)\left(1-\frac{2}{\beta^2}\right)\right]\nabla^2 q-f(T)(1-b^*\nabla^2)\,\phi=\left[\left(1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}\right)\ddot{q}-q\Omega^2-2\Omega\dot{\psi}\right], \tag{20}$$

$$(a_1\nabla^2+\Omega^2)\psi=\left[\left(1+\frac{\varepsilon_0\mu_0^2 H_0^2}{\rho}\right)\ddot{\psi}+2\Omega\dot{q}\right], \tag{21}$$

$$\varepsilon_1\nabla^2\phi+\varepsilon_2\nabla^2\dot{\phi}+\tau_t\nabla^2\ddot{\phi}=\left(1+\tau_q\frac{\partial}{\partial t}+\frac{\tau_q^2}{2!}\frac{\partial^2}{\partial t^2}\right)\left[(1-b^*\nabla^2)\ddot{\phi}+\varepsilon_3\nabla^2\ddot{q}\right]. \tag{22}$$

3. Normal mode analysis

The solution of the considered physical variable can be decomposed in terms of normal modes as the following form:

$$[u, v, e, T, \phi, q, \psi, \sigma_{ij}](x, y, t)=\left[u^*, v^*, e^*, T^*, \phi^*, q^*, \psi^*, \sigma_{ij}^*\right](y)\exp[i(\omega t+ax)], \tag{23}$$

where ω is the complex time constant and a is the wave number in x-direction.

Using (23) in Equations (20)-(22), we obtain:

$$\left(A_1 D^2 + A_2\right) q^* + \left(b^* D^2 - A_3\right) f(T) \phi^* + A_4 \psi^* = 0, \tag{24}$$

$$\left(a_1 D^2 + A_5\right) \psi^* - A_4 q^* = 0, \tag{25}$$

$$\left(A_6 D^2 - A_7\right) \phi^* + A_8 \left(D^2 - a^2\right) q^* = 0, \tag{26}$$

where:

$$A_1 = \frac{\mu_0 H_0^2}{\rho c_0^2} + 2a_1 + \left(1 - \frac{2}{\beta^2}\right) f(T),$$

$$A_2 = -2a_1 a^2 - a^2 \left(1 - \frac{2}{\beta^2}\right) - \frac{a^2 \mu_0 H_0^2}{\rho c_0^2} + \left[1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho}\right] \omega^2 + \Omega^2,$$

$$A_3 = 1 + b^* a^2,\ A_4 = 2i\Omega\omega,\ A_5 = -a_2 a^2 + \Omega^2 + \left[1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho}\right] \omega^2,$$

$$A_6 = \varepsilon_1 + i\varepsilon_2 \omega - \tau_t \omega^2 - b^* \omega^2 \left[1 + i\tau_q \omega - \frac{\tau_q^2}{2!} \omega^2\right],$$

$$A_7 = \varepsilon_1 a^2 + i\varepsilon_2 \omega a^2 - \tau_t \omega^2 a^2 - (1 + b^* a^2) \omega^2 \left[1 + i\tau_q \omega - \frac{\tau_q^2}{2!} \omega^2\right],$$

$$A_8 = \varepsilon_3 \omega^2 \left[1 + i\tau_q \omega - \frac{\tau_q^2}{2!} \omega^2\right].$$

Eliminating ϕ^* and ψ^* between Equations (24)-(26), we get:

$$\left[D^6 - AD^4 + BD^2 - C\right] \{q^*(y),\ \phi^*(y),\ \psi^*(y)\} = 0. \tag{27}$$

Where:

$$A = \frac{-\ A_1 A_7 a_1 + A_1 A_5 A_6 + A_2 A_6 a_1 + f(T) b^* a^2 a_1 A_8 - f(T) A_5 A_8 b^* + f(T) a_1 A_3 A_8}{f(T) a_1 A_8 b^* - a_1 A_1 A_6}$$

$$B = \frac{a_1 A_2 A_7 - A_2 A_5 A_6 - f(T) b^* A_8 A_5 a^2 + f(T) A_3 A_8 a_1 a^2 - f(T) A_3 A_5 A_8 - A_4^2 A_6 + A_1 A_5 A_7}{f(T) a_1 A_8 b^* - a_1 A_1 A_6}$$

$$C = \frac{-A_2 A_5 A_7 - f(T) A_3 A_5 A_8 a^2 - A_4^2 A_7}{f(T) a_1 A_8 b^* - a_1 A_1 A_6} \quad D = \frac{d}{dy}.$$

The solution of Equation (27) has the form:

$$q^* = \sum_{n=1}^{3} M_n e^{-k_n y}, \tag{28}$$

$$\phi^* = \sum_{n=1}^{3} H_{1n} M_n e^{-k_n y}, \tag{29}$$

$$\psi^* = \sum_{n=1}^{3} H_{2n} M_n e^{-k_n y}, \tag{30}$$

$$T^* = \sum_{n=1}^{3} H_{3n} M_n e^{-k_n y}, \tag{31}$$

where M_n ($n=1, 2, 3$) are some constants, k_n^2 are the roots of the characteristic equation of Equation (27).

Dimensionless variables of the stress components yield the following:

$$\sigma_{xx} = f(T)u_{,x} + \left(1 - \frac{2}{\beta^2}\right) f(T)v_{,y} - f(T)T, \tag{32}$$

$$\sigma_{yy} = f(T)v_{,y} + \left(1 - \frac{2}{\beta^2}\right) f(T)u_{,x} - f(T)T, \tag{33}$$

$$\sigma_{xy} = \frac{\mu_0 f(T)}{\rho c_0^2}\left[u_{,y} + v_{,x}\right]. \tag{34}$$

Using Equation (19) and Equations (28)-(31) in (32)-(34) we get:

$$u = \sum_{n=1}^{3} H_{4n} M_n e^{i(\omega t + ax) - k_n y}, \tag{35}$$

$$v = \sum_{n=1}^{3} H_{5n} M_n e^{i(\omega t + ax) - k_n y}, \tag{36}$$

$$\sigma_{xx} = \sum_{n=1}^{3} H_{6n} M_n e^{i(\omega t + ax) - k_n y}, \tag{37}$$

$$\sigma_{yy} = \sum_{n=1}^{3} H_{7n} M_n e^{i(\omega t + ax) - k_n y}, \tag{38}$$

$$\sigma_{xy} = \sum_{n=1}^{3} H_{8n} M_n e^{i(\omega t + ax) - k_n y}, \tag{39}$$

$$T = \sum_{n=1}^{3} H_{3n} M_n e^{i(\omega t + ax) - k_n y}. \tag{40}$$

where:

$$H_{1n} = -\frac{A_8\left(k_n^2 - a^2\right)}{A_6 k_n^2 - A_7},\ H_{2n} = \frac{A_4}{A_5 + a_1 k_n^2},\ H_{3n} = H_{1n}\left[1 - b^*\left(k_n^2 - a^2\right)\right],\ H_{4n} = (ia + k_n H_{2n}),$$

$$H_{5n} = (-k_n + iaH_{2n}),\ H_{6n} = \left[iaH_{4n} f(T) - \left(1 - \frac{2}{\beta^2}\right) f(T) k_n H_{5n} - f(T) H_{3n}\right],$$

$$H_{7n} = \left[-f(T) k_n H_{5n} + iaH_{4n}\left(1 - \frac{2}{\beta^2}\right) f(T) - f(T) H_{3n}\right],\ H_{8n} = (-a_1 k_n H_{4n} + iaa_1 H_{5n}).$$

4. Boundary conditions

The boundary conditions on the plane surface $y = 0$ are:

$$\sigma_{xx} = P_1 e^{i(\omega t + ax)},\ \sigma_{xy} = 0,\ T = P_2 e^{i(\omega t + ax)}. \tag{41}$$

Using Equations (37), (39) and (40) in the boundary conditions (41), we get three equations in three constants M_n ($n = 1, 2, 3$) as:

$$\sum_{n=1}^{3} H_{6n} M_n = P_1, \tag{42}$$

$$\sum_{n=1}^{3} H_{8n} M_n = 0, \tag{43}$$

$$\sum_{n=1}^{3} H_{3n} M_n = P_2. \tag{44}$$

Solving Equations (42)-(44), the constants M_n ($n = 1, 2, 3$) are defined as follows:

$$M_1 = \frac{\Delta_1}{\Delta},\ M_2 = \frac{\Delta_2}{\Delta},\ M_3 = \frac{\Delta_3}{\Delta}.$$

where $\Delta = H_{61}(H_{83} H_{32} - H_{82} H_{33}) - H_{62}(H_{83} H_{31} - H_{81} H_{33}) + H_{63}(H_{82} H_{31} - H_{81} H_{32})$
$\Delta_1 = P_1(H_{83} H_{32} - H_{82} H_{33}) - P_2(H_{83} H_{62} - H_{63} H_{82})$, $\Delta_2 = -P_1(H_{83} H_{31} - H_{81} H_{33}) + P_2(H_{61} H_{83} - H_{63} H_{81})$, $\Delta_3 = P_1(H_{82} H_{31} - H_{81} H_{32}) + P_2(H_{62} H_{81} - H_{61} H_{82})$.

5. Numerical results

The copper material was chosen for purposes of numerical evaluations. The material constants of the problem were then taken as Roy Choudhuri (2007):

$$\lambda = 7.7 \times 10^{10}\ \text{N.M}^{-2},\ \mu = 3.86 \times 10^{10}\ \text{Kg.m}^{-1}.\text{s}^{-2},\ K = 300\ \text{w.m}^{-1}.\text{K}^{-1},\ K^* = 2.97 \times 10\text{\^{}}13,\ x = 0.1,$$

$$\alpha_t = 1.78 \times 10^{-5}\ \text{K}^{-1},\ \rho = 8{,}954\ \text{Kg.m}^{-3},\ t = 0.1,$$

$$c_e = 383.1\ \text{J.Kg}^{-1}\text{K}^{-1},\ \tau_q = 0.8,\ T_0 = 293\,K,$$

$$P_1 = -0.001,\ P_2 = -0.002,\ \ b = 0.1,\ \ \Omega = 0.2,\ H_0 = 10^3,$$

$$a = 1.000015,\ \ \tau_t = 0.6,\ \ \tau_v = 0.2, \omega = -4.000000000892111.$$

Figures 1-6 show the comparisons between physical quantities based on the DPL and 3PHL models in the case of a material with temperature-dependent properties ($\alpha^* = 0.0001517$).

Figure 1 depicts that the displacement component u increases in the case of $\alpha^* = 0$ under the 3PHL model and increases in the case of $\alpha^* = 0.0001517$ under DPL model then decreases until it develops to 0. The displacement component u increases in the case of $\alpha^* = 0.0001517$ under the 3PHL model and increases in the case of $\alpha^* = 0$ under the DPL model, then decreases until it develops to 0. Figure 2 shows that the displacement component v decreases, then increases until it develops to 0 in the case of $\alpha^* = 0$ and $\alpha^* = 0.0001517$ under the 3PHL and DPL models. Figure 3 demonstrates that the behavior of temperature T increases, then decreases to a minimum until it develops to 0 in the case of $\alpha^* = 0$ and $\alpha^* = 0.0001517$ under the 3PHL and DPL models. Figure 4 explains that the stress component σ_{xx} decreases to the minimum point between $0 \leqslant y \leqslant 1$, in the two models, and increases in the range $1 \leqslant y \leqslant 4$, in the two models then decays to 0. Figure 5 depicts that the stress component σ_{yy} increase to the maximum point between $0 \leqslant y \leqslant 2$, in the two

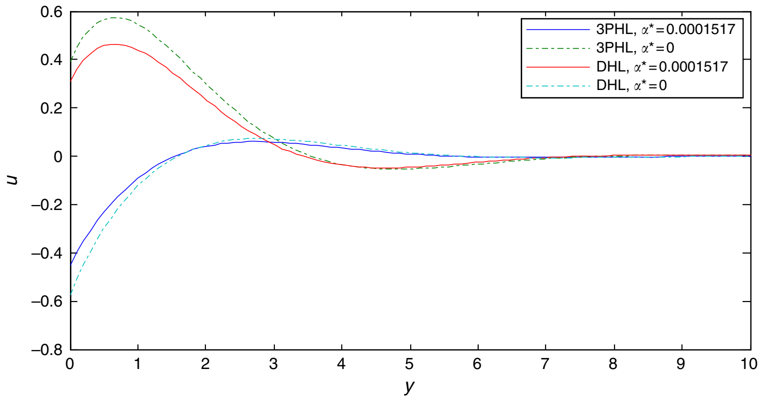

Figure 1.
Effect of α^* on displacement component u

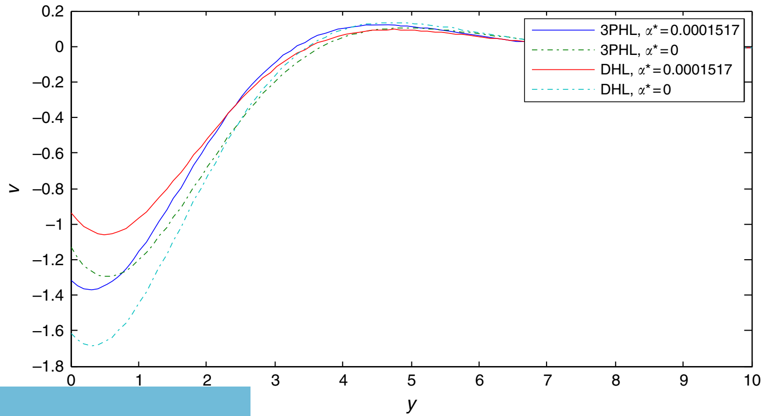

Figure 2.
Effect of α^* on displacement component v

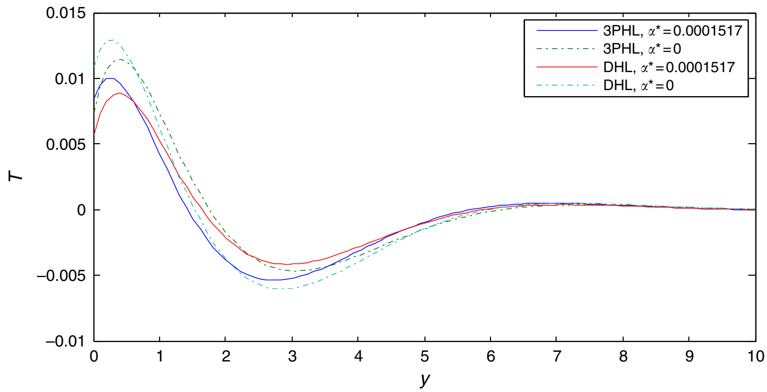

Figure 3.
Effect of α^* on temperature T

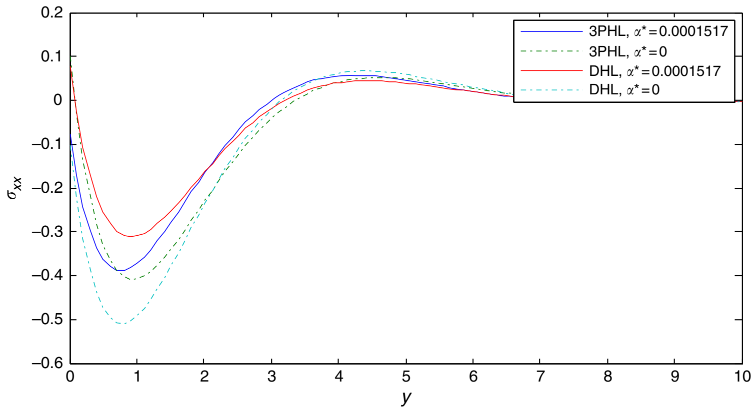

Figure 4.
Effect of α^* on stress component σ_{xx}

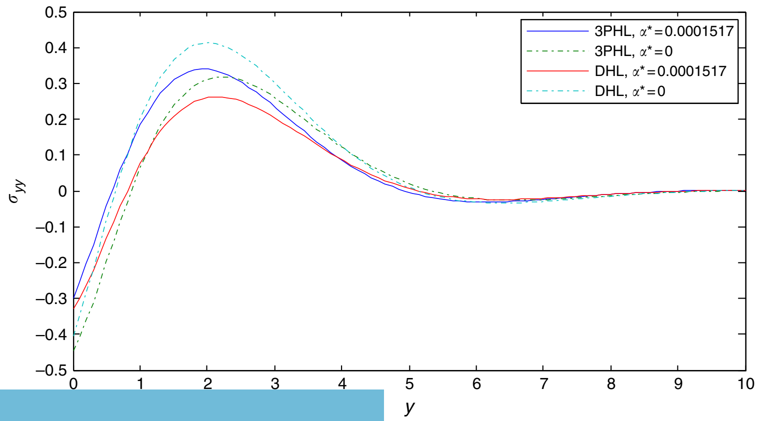

Figure 5.
Effect of α^* on stress component σ_{yy}

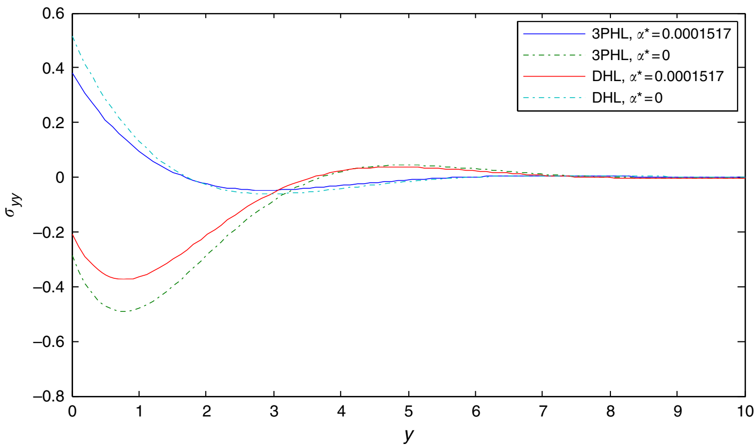

Figure 6.
Effect of α^* on stress component σ_{xy}

models, and decreases in the range $2 \leqslant y \leqslant 5$, in the two models then decays to 0. Figure 6 shows that the stress component σ_{xy} decreases in the case of $\alpha^* = 0$ and $\alpha^* = 0.0001517$ in the two models, then increases in the case of $\alpha^* = 0.0001517$ under the DPL model and increases in the case of $\alpha^* = 0$ under the 3PHL models until it develops to 0.

6. Conclusion

Analytical solutions based upon normal mode analysis of the thermoelastic problem in solids have been developed. The method that is used in the present paper is applicable to a wide range of problems in hydrodynamics and thermoelasticity. There are significant differences in the field quantities under the DPL and 3PHL models. The presence of magnetic field, rotation and two-temperature plays a significant role in all the physical quantities. The presence of temperature-dependent properties plays an important role in all physical quantities except the temperature. The amplitude of the physical quantities changes in the case of temperature-dependent properties.

Deformation of a body depends on the nature of force applied as well as the type of boundary conditions. The comparison of the two models of thermoelasticity, DPL and 3PHL is carried out. The value of all the physical quantities converges to 0 and all the functions are continuous.

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About the authors

Mohamed I.A. Othman was born on September 5, 1958. In 1994, he received PhD Degree in Mathematics from the Zagazig University, Egypt. His nationality is Egyptian. He is a Member of the Egyptian Mathematical Society and IBC Research Council. At present, his is affiliated to the Taif University, Saudi Arabia. He works in the field of Theory of Thermoelasticity. His most important papers are Theory of Two-Temperature Generalized Thermoelasticity, Thermoelastic Diffusion, Fiber-reinforced, Thermoelastic with Voids, Micropolar Thermoelastic Medium, Piezo-Thermoelastic and Generalized Thermo-Microstretch Elastic Solid. He is the owner of a school of 25 master and doctoral students. He has reviewed more than 65 international journals. His more detailed CV can be found in "Who's Who in the Thermal-Fluid, 2011", "Who's Who in Science and Engineering", 2012 and in "Encyclopedia of Thermal Stresses 2011." He has published 220 papers in international journals. His total citations are 2,366 till September 24, 2016 as per the i10-index 60 h-index 32. Mohamed I.A. Othman is the corresponding author and can be contacted at: m_i_a_othman@yahoo.com

Yassmin D. Elmaklizi is a Dr in the Faculty of Science, Suez Canal University, Suez, Egypt. Her most important papers are "Theory of two-temperature generalized thermoelasticity, Thermo-elastic diffusion, and Thermoelastic with voids". She published 25 papers in international journals.

Nehal T. Mansoure was born on September 1, 1988. She received Graduation Degree from the Department of Mathematics, Faculty of Science, in 2009. She got a Diploma in Education in 2011. She received a Master's Degree in Applied Mathematics (Thermoelasticity) in 2015. Her most important papers are "Theory of two-temperature generalized thermoelasticity, Thermo-elastic diffusion, and Thermoelastic with voids". She published 15 papers in international journals.